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THE ACTION OF A CONCENTRATED FORCE INSIDE AN ELASTIC HALF-PLANE WITH A STEP†

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A solution of the plane problem in the theory of elasticity of the stressed state of a half-plane with a step, at an arbitrary internal point of which a concentrated force is applied, is presented. The solution is constructed by the method of Cauchy-type integrals, developed by Muskhelishvili. A Koppenfels function is used for the conformal mapping of the half-plane with a step into a half-plane. Graphs of the distribution of the maximum shear stresses in the neighbourhood of the point of application of the force and the trajectories of the principal tensile stresses are presented. Being exact and expressed in terms of elementary functions, the solution presented can be used as a test problem to debug numerical solutions as well as a component part of software packages for computer aided calculations and the optimization of constructive solutions of supports, sunk into inclines.

The problem of the stress–strain state of a half-plane with a step when subjected to an internal concentrated force arises in calculations of the strength of supports, sunk into the ground, of floating hydrotechnical installations which are kept floating using cables fixed on the coastal incline by horizontally extended anchors and, also, of extended anchor mountings of cable cranes, suspended cable ways, suspension bridges, aerial networks and other installations on slopes and inclines.

In the case of a homogeneous ground, the model of a linearly deformable medium can be used to determine the stress–strain state of inclines.

The solution of the problem of the stressed state of an elastic half-plane with a rectilinear boundary (without a step), at an internal point of which a concentrated load is imposed on the imaginary axis, was constructed for the first time by Melan [1].

1. FORMULATION OF THE PROBLEM

A statically applied concentrated force $\mathbf{F}_z = F_x + iF_y$ (i is the square root of -1) acts, as shown in Fig. 1, at a point $z_0 = x_0 + iy_0$ in a half-plane with a step of height h with an angle of inclination πc ($c = p/q$, where p and q are integers and $p < q$). There are no loads on the external faces of the half-plane with a step. The material of the half-plane is characterized by its Young's modulus E and Poisson's ratio ν. It is required to find the stress distribution in the half-plane with a step.

The solution of this problem will obviously also hold for a problem involving the action in an elastic half-space with a terrace of a load $\mathbf{F}_z$ which is distributed along a line perpendicular to the xOy plane. In this case, there will not be a plane stressed state but plane deformation.

Note that the mirror image of the upper half-plane with a step in the x axis is shown in Fig. 1, since the conformal mapping $z = \omega(\zeta)$ of the upper half-plane with a step into the upper half-plane ζ is used in the solution presented below. Apart from the boundary of the step (the heavy dashed line in xO, yO coordinates), several coordinate lines η = const of the curvilinear system of coordinates are shown in Fig. 1.

2. METHOD OF SOLUTION

The problem has an exact analytical solution which can be constructed by the method of Cauchy-type integrals [2].

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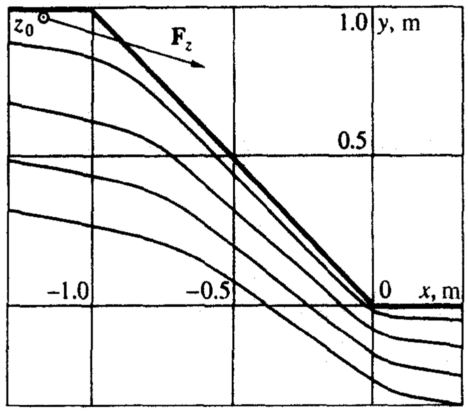

Fig. 1

We conformally map the domain z of the step into the half-plane ζ by means of the functions

$$\omega(\zeta) = \frac{h}{\pi}\left[\frac{s^p}{c(1-s^q)} + \sum_{n=0}^{q-1} t_n^{p-q} \ln\left(1 - \frac{s}{t_n}\right)\right] \tag{2.1}$$

$$s = (1-\zeta^{-1})^{1/q}, \quad \zeta > 0;$$

$$t_n = \exp(2n\pi i/q), \quad n = 0, 1, 2, \ldots, q-1$$

(s is a function which maps the angular domain into the half-plane and t_n are the roots of the nth power of unity).

Function (2.1) was obtained for the first time [3] using the Christoffel – Schwarz formula for rectilinear polygons. However, an inaccuracy slipped into the expression for this function. Later, on the basis of [3], function (2.1) was constructed again [4] and the inaccuracy was removed.

The mapping (2.1) can be considered as the introduction into the z plane of a curvilinear system of coordinates $\xi = \xi(x, y)$, $\eta = \eta(x, y)$:

$$z = x + iy, \quad \zeta = \xi + i\eta, \quad \zeta = \Omega(z), \quad \xi = \operatorname{Re}\Omega(z), \quad \eta = \operatorname{Im}\Omega(z)$$

where $\Omega(z)$ is a function which is the inverse of $\omega(\zeta)$.

It is not possible to construct an expression for $\Omega(z)$ in terms of elementary functions and it is therefore henceforth assumed that the function $\Omega(z)$ is specified in tabulated form.

On the whole, the problem will be solved if the two complex potentials $\Phi(\zeta)$ and $\Psi(\zeta)$ are found which satisfy the boundary condition on the free surface of the step when $\eta = 0$.

The relation $\zeta_0 = \Omega(z_0)$ corresponds to the application of a force $\mathbf{F}_z$ in the ζ plane at the point z_0, and the expression [2]

$$\mathbf{F}_\zeta = F_\xi + iF_\eta = \frac{\overline{\omega'(\zeta)}}{|\omega'(\zeta)|}(F_x + iF_y) \tag{2.2}$$

corresponds to the force itself.

The solution of the problem for a concentrated force in the whole z plane is known [2]; in the curvilinear system of coordinates (ξ, η), this solution has the form

$$\Phi_1(\zeta) = \frac{-P_\zeta}{\omega(\zeta) - z_0}, \quad \Psi_1(\zeta) = \frac{\varkappa\overline{\mathbf{P}}_\zeta}{\omega(\zeta) - z_0} - \frac{\bar{z}_0\overline{\mathbf{P}}_\zeta}{(\omega(\zeta) - z_0)^2} \tag{2.3}$$

$$\mathbf{P}_\zeta = \frac{F_\xi + iF_\eta}{2\pi(1+\varkappa)}, \quad \varkappa = \begin{cases}(1-\nu)/(1+\nu)\\ 3-4\nu\end{cases}$$

(the coefficient $\varkappa$ takes the value in the upper row in the case of a plane stressed state and the value in the lower row in the case of plane deformation).

We will henceforth omit the index ζ in the notation for the force $\mathbf{P}$.

The stresses σ_ξ, σ_η, $\tau_{\xi\eta}$ in the curvilinear system of coordinates (ξ, η) are given by the Kolosov–Muskhelishvili formulae [2, 5], from which follows an expression for the stress vector $\sigma_\eta + i\tau_{\xi\eta}$, acting on the coordinate line η = const, and for the vector conjugate to it

$$\sigma_\eta + i\tau_{\xi\eta} = A_1(\zeta) + B_1(\zeta), \quad \sigma_\eta - i\tau_{\xi\eta} = A_1(\zeta) + \overline{B_1(\zeta)} \tag{2.4}$$

$$A_1(\zeta) = \Phi_1(\zeta) + \overline{\Phi_1(\zeta)}, \quad B_1(\zeta) = \frac{1}{\overline{\omega'(\zeta)}}[\overline{\omega(\zeta)}\Phi_1'(\zeta) + \omega'(\zeta)\Psi_1(\zeta)]$$

The concentrated force, applied at the point $\zeta_0 = \Omega(z_0)$, creates stresses on the surface of the step $\eta = 0$ which can be determined using expressions (2.4) when $\zeta \to t = \xi + 0i$, that is, $\eta \to 0$,

$$\sigma_\eta^{(0)} + i\tau_{\xi\eta}^{(0)} = A_1(t) + B_1(t), \quad \sigma_\eta^{(0)} - i\tau_{\xi\eta}^{(0)} = A_1(t) + \overline{B_1(t)} \tag{2.5}$$

In order to solve the problem, it is necessary to "remove" the stresses on the line $\eta = 0$ $\sigma_\eta^{(0)}$, $\tau_{\xi\eta}^{(0)}$, which are determined by one of the formulae (2.5), that is, to find the potentials $\Phi_*(\zeta)$, $\Psi_*(\zeta)$ which cause stresses $-(\sigma_\eta^{(0)} + i\tau_{\xi\eta}^{(0)})$ or $-(\sigma_\eta^{(0)} - i\tau_{\xi\eta}^{(0)})$ on the line $\eta = 0$.

The solution of the problem will then be found as a superposition of the potentials Φ_1, Ψ_1 and Φ_*, Ψ_*.

We find the auxiliary potentials $\Phi_*(\zeta)$ and $\Psi_*(\zeta)$ from the boundary conditions corresponding to expressions (2.5)

$$A_*(t) + B_*(t) = -(\sigma_\eta^{(0)} + i\tau_{\xi\eta}^{(0)}), \quad A_*(t) + \overline{B_*(t)} = -(\sigma_\eta^{(0)} - i\tau_{\xi\eta}^{(0)}) \tag{2.6}$$

(the expressions for the functions $A_*(t)$ and $B_*(t)$ are obtained from the expressions for the functions $A_1(t)$ and $B_1(t)$ (2.4) by replacing the subscripts 1 in their right-hand sides by the subscripts *).

On multiplying each term of the second expression of (2.6) by $1/[2\pi i(t-\zeta)]$ and integrating between infinite limits along the line $\eta = 0$, we obtain a sum of Cauchy-type integrals

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\sigma_\eta^{(0)} - i\tau_{\xi\eta}^{(0)}}{t-\zeta}dt + \frac{1}{2\pi i}\int_{-\infty}^{\infty}[A_*(t) + \overline{B_*(t)}]\frac{dt}{t-\zeta} = 0 \tag{2.7}$$

where ζ is a point of the half-plane $\eta > 0$.

Assuming that $\Phi_*(t)$ is the boundary value of the required function $\Phi_*(\zeta)$, which is homomorphic when $\eta > 0$ and vanishes at infinity and that $A_*(t)$ and $\overline{B_*(t)}$ are the boundary values of the functions $A_*(\zeta)$ and $\overline{B_*(\zeta)}$, which are homomorphic when $\eta < 0$ and vanish at infinity, we obtain from relation (2.7), using Cauchy's formula and its corollaries for the half-plane $\eta > 0$,

$$\Phi_*(\zeta) = -\frac{\varkappa \mathbf{P}}{\omega(\zeta) - \bar{z}_0} + \frac{(z_0 - \bar{z}_0)\overline{\mathbf{P}}}{(\omega(\zeta) - \bar{z}_0)^2} \tag{2.8}$$

(the second expression of (2.4) and expressions (2.3) have been taken into account).

In order to construct the function $\Psi_*(\zeta)$, we use the first boundary condition of (2.6) and rewrite it in the form

$$\Phi_*(t) + \overline{\Phi_*(t)} + \frac{\overline{\omega(t)}}{\overline{\omega'(t)}}\Phi_*'(t) + \frac{\omega'(t)}{\overline{\omega'(t)}}\Psi_*(t) + (\sigma_\eta^{(0)} + i\tau_{\xi\eta}^{(0)}) = 0 \tag{2.9}$$

Multiplying each term of (2.9) by $1/[2\pi i(t-\zeta)]$ and integrating between infinite limits along the line $\eta = 0$, we obtain a sum of Cauchy-type integrals and, from Cauchy's formula and its corollaries for the half-plane $\eta > 0$, the fourth term on the left-hand side of this sum is expressed by the homomorphic function

$$\frac{\omega'(\zeta)}{\overline{\omega}'(\zeta)}\Psi_*(\zeta) = -\frac{1}{2\pi i}\left[\int_{-\infty}^{\infty}\frac{\sigma_\eta^{(0)} + i\tau_{\xi\eta}^{(0)}}{t-\zeta}dt + \int_{-\infty}^{\infty}\left[A_*(t) + \frac{\overline{\omega(t)}}{\overline{\omega'(t)}}\Phi_*'(t)\right]\frac{dt}{t-\zeta}\right] \tag{2.10}$$

We also calculate the right-hand side or relation (2.10) as in the case of the function $\Phi_*(\zeta)$. The factor $\omega'(\zeta)/\overline{\omega'(\zeta)}$ on the left-hand side of relation (2.10) when $\zeta \to t$ is equal to unity.

In fact, differentiating the mapped function (2.1) with respect to ζ, we obtain an expression for $\omega'(\zeta)$ which, when $\zeta \to t$, becomes a real function, since it either preserves its form or is supplemented by complex-conjugate terms in a pairwise manner, which give a real function in the sum.

For example, for steps with angles of inclination $\pi c = \pi/2, \pi/3, \pi/4$ and $\pi/6$ ($p = 1$), we have

$$\pi c = \frac{\pi}{2}, \quad s = (1-\zeta^{-1})^{1/2}, \quad \omega'(\zeta) = \frac{h}{\pi s}(1+s+s^2+s^3) \tag{2.11}$$

$$\pi c = \frac{\pi}{3}, \quad s = (1-\zeta^{-1})^{1/3}, \quad \omega'(\zeta) = \frac{h}{\pi\zeta^2}\left[2\zeta^2 - \zeta + \frac{1}{1-s} + \frac{2-s}{1-s+s^2}\right]\left(\frac{\zeta}{\zeta-1}\right)^{2/3} \tag{2.12}$$

$$\pi c = \frac{\pi}{4}, \quad s = (1-\zeta^{-1})^{1/4}, \quad \omega'(\zeta) = \frac{4h}{\pi}s \tag{2.13}$$

$$\pi c = \frac{\pi}{6}, \quad s = (1-\zeta^{-1})^{1/6}, \quad \omega'(\zeta) = \frac{h}{6\pi\zeta^2}\left(12\zeta^2 - \zeta - \frac{1}{s} - \frac{1}{1-\bar{a}s} - \frac{1}{1+as} - \frac{1}{1+s} - \frac{1}{1+\bar{a}s} - \frac{1}{1-as}\right)\times$$

$$\times\left(\frac{\zeta}{\zeta-1}\right)^{5/6}, \quad a = 0.5 + 0.866i \tag{2.14}$$

Functions (2.11)–(2.13) retain their form on putting $\zeta = t$, becoming real, and function (2.14) becomes real by virtue of the fact that the fourth and eighth terms, as well as the fifth and seventh terms on the right-hand side are pairwise complex-conjugate. Hence, if p and q are integers and $p < q$, then $\overline{\omega'(t)} = \omega(t)$.

Taking the equality $\overline{\omega'(t)} = \omega(t)$ into account, from relation (2.10), we find

$$\Psi_*(\zeta) = -\frac{\varkappa\bar{z}_0}{(\omega(\zeta)-\bar{z}_0)^2}\mathbf{P} + \left[\frac{\omega(\zeta)-z_0}{(\omega(\zeta)-\bar{z}_0)^2} + \frac{2(z_0-\bar{z}_0)\omega(\zeta)}{(\omega(\zeta)-\bar{z}_0)^3}\right]\overline{\mathbf{P}}$$

The superposition of the potentials Φ_1 and Φ_*, Ψ_1 and Ψ_* gives the solution of the problem

$$\Phi(\zeta) = -\left(\frac{\varkappa}{\omega(\zeta)-\bar{z}_0} + \frac{1}{\omega(\zeta)-z_0}\right)\mathbf{P} + \frac{z_0-\bar{z}_0}{(\omega(\zeta)-\bar{z}_0)^2}\overline{\mathbf{P}} \tag{2.15}$$

$$\Psi(\zeta) = -\bar{z}_0\left[\frac{\varkappa}{(\omega(\zeta)-\bar{z}_0)^2} + \frac{1}{(\omega(\zeta)-z_0)^2}\right]\mathbf{P} +$$

$$+\left[\frac{\varkappa}{\omega(\zeta)-z_0} + \frac{\omega(\zeta)-z_0}{(\omega(\zeta)-\bar{z}_0)^2} + \frac{2(z_0-\bar{z}_0)\omega(\zeta)}{(\omega(\zeta)-\bar{z}_0)^3}\right]\overline{\mathbf{P}} \tag{2.16}$$

Substitution of expressions (2.15) and (2.16) when $\zeta \to t$ into the first boundary condition of (2.5) makes its right-hand side vanish, which confirms the correctness of the solution.

3. THE STRESS DISTRIBUTION

The stressed state of the step in the curvilinear system of coordinates (ξ, η) is given by the formulae

$$A(\zeta) = \Phi(\zeta) + \overline{\Phi(\zeta)}, \quad B(\zeta) = \frac{1}{\overline{\omega'(\zeta)}}[\overline{\omega(\zeta)}\Phi'(\zeta) + \omega'(\zeta)\Psi(\zeta)]$$

$$\sigma_\xi = \mathrm{Re}[A(\zeta) - B(\zeta)], \quad \sigma_\eta = \mathrm{Re}[A(\zeta) + B(\zeta)], \quad \tau_{\xi\eta} = \mathrm{Im}\, B(\zeta) \tag{3.1}$$

Using the Kolosov–Muskhelishvili formulae, the principal stresses σ_1 and σ_2, the maximum shear

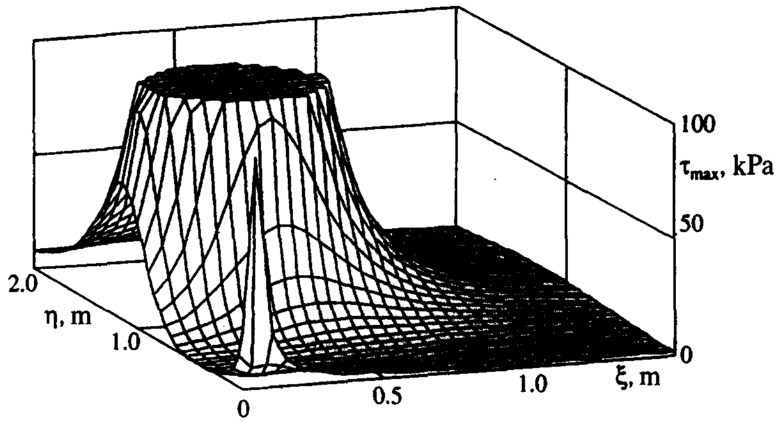

Fig. 2

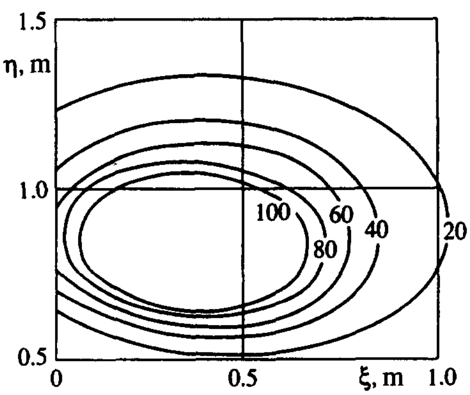

Fig. 3

stress τ_{max}, the intensity of the normal stresses σ_i, and the octahedral normal stresses σ_{oct} and shear stresses τ_{oct} can be expressed in terms of the potentials $\Phi(\zeta)$ and $\Psi(\zeta)$ as follows:

$$\sigma_1 = A(\zeta) + |B(\zeta)|, \quad \sigma_2 = A(\zeta) - |B(\zeta)|, \quad \tau_{max} = |B(\zeta)|$$
$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2} \tag{3.2}$$
$$\sigma_{oct} = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3), \quad \tau_{oct} = \frac{\sqrt{2}}{3}\sigma_i$$

In the case of plane strain, the third principal stress is defined as $\sigma_3 = \nu(\sigma_1 + \sigma_2)$.

It can be seen that the stress fields σ_i and τ_{oct} are similar to one another; by substituting $\sigma_3 = \nu(\sigma_1 + \sigma_2)$ into the fourth expression of (3.2), it can be shown that the stress fields σ_i and τ_{max} are extremely close to one another and that the field σ_{oct} is similar to the field σ_3.

To construct any of the stress fields, it is necessary to introduce a mesh of rectangular coordinates ξ_k, η_m in the half-plane $\zeta > 0$ which, by means of the function (2.1), is transformed into the mesh of curvilinear coordinates $x_{k,m} = \mathrm{Re}\,\omega(\xi_k, \eta_m)$, $y_{k,m} = \mathrm{Im}\,\omega(\xi_k, \eta_m)$ on the step. At the mesh points ξ_k, η_m in the half-plane $\zeta > 0$, it is possible, using expressions (2.15) and (2.16), to calculate the potentials Φ and Ψ, and all of the stresses using expressions (3.1) and (3.2). The calculated stresses act at the mesh points of the mesh $x_{k,m}$, $y_{k,m}$ of the curvilinear coordinates of the step.

The truncated spatial graph of the distribution of the maximum shear stresses τ_{max} is shown in Fig. 2 in the local rectangular system of coordinates for the following initial data: height of step $h = 1$ m, angle of inclination of the face $\pi c = \pi/4$ ($p = 1$, $q = 4$), Poisson's ratio of the step material $\nu = 0.25$, the concentrated force has the components $F_x = 20$ kN, $F_y = 5$ kN, and the coordinates of the point of application of the force is $x_0 = -1.191$ m, $y_0 = -0.881$ m. The point $\zeta_0 = -0.05 + 0.05i$ corresponds to the point of application of the force $z_0 = -1.191-0.881i$ in the ζ plane and the force $\mathbf{F}_\zeta = 1.092 +$

0.066i corresponds to the concentrated force $\mathbf{F}_z = 1.061 + 0.265i$. The graph of τ_{max} is truncated using the arbitrary quantity τ_{10} which is equal to ten times the mean value of the maximum shear stresses at the mesh points of the calculated domain of the step: $\tau_{10} = 10\tilde{\tau}_{max}$, $\tilde{\tau}_{max}$ is the mean value. At those points of the step where the stresses τ_{max} exceeds the values τ_{10} they are equated to these values.

A contour map of the levels of the maximum shear stresses τ_{max}, constructed in the ξ, η plane in the neighbourhood of the point of application of the force $\mathbf{F}_z$, is shown in Fig. 3. The lines of equal values are numbered in units of kPa. The contour map of the levels enables one to estimate the position and dimensions of the zone of stress concentration.

A combined consideration of graphs 2 and 3 enables us to conclude that the concentrated force has a substantial effect on the stressed state of the step in the neighbourhood of its point of application, giving rise to a stress concentration in the immediate vicinity of the point of application and, also, in the neighbourhood of the upper corner point. The zone of the effect of the concentrated force is stretched in the direction of the action of the force. The size of the zone of influence depends on the magnitude of the concentrated force, the coordinates of its point of application, the geometry of the step (the height and angle of inclination of the face) and, also, on Poisson's ratio of the step material.

4. TRAJECTORIES OF THE PRINCIPAL STRESSES

As an example of the use of the solution, we will construct the trajectories of the principal stresses in the step due to the action of an internal concentrated force. The pattern of the trajectories of the principal stresses (that is, of the lines, the tangents of which coincide with the directions of the principal stresses) gives a clear representation of the structure of the stressed state, allowing one, for example, to indicate the directions of possible cracks.

In the plane problem, the direction cosines $n_1^{(k)}$, $n_2^{(k)}$ of the normals to the planes of the principal stresses are determined by the system of equations

$$(\sigma_\xi - \sigma_k)n_1^{(k)} + \tau_{\xi\eta}n_2^{(k)} = 0, \quad \tau_{\xi\eta}n_1^{(k)} + (\sigma_\xi - \sigma_k)n_2^{(k)} = 0$$
$$[n_1^{(k)}]^2 + [n_2^{(k)}]^2 = 1; \quad k = 1,2 \tag{4.1}$$

If it is assumed that $n_1^{(k)} = \cos\alpha_k$, $n_2^{(k)} = \sin\alpha_k$, then each of the first two equations of (4.1) gives an expression for tg α_k. Since tg $\alpha_k = d\eta_k/d\xi$, we have the first-order differential equation

$$d\eta_k / d\xi = (\sigma_k - \sigma_\xi)/\tau_{\xi\eta} = \tau_{\xi\eta}/(\sigma_k - \sigma_\eta), \quad k = 1,2 \tag{4.2}$$

for constructing the trajectories of the principal stresses.

By specifying the initial values $\eta_k^{(0)}$ on the coordinate line $\xi = \xi_0$, it is possible to construct the integral curves of Eq. (4.2), starting out from the points ξ_0, $\eta_k^{(0)}$. By changing the right-hand side of Eq. (4.2) during the integration, it is possible to monitor the correctness of the construction of the trajectories of the principal stresses.

The right-hand side of Eq. (4.2) is a complex function of a complex variable which has large gradients close to singular points and, because of this, it is impossible to use standard programs for numerical integration.

We wrote a program for integrating this equation in which the initial segment of the integral curve is constructed by a fourth-order Runge–Kutta method with a variable step size [6, 7] modified by Merson [8]; the remaining part of the integral curve is found using the "predictor–corrector" method [7, 8] which is also of the fourth-order of accuracy with a variable step size.

The trajectories of the principal tensile stresses σ_1, constructed using the same initial data as for the graphs in Figs 2 and 3, are presented in Fig. 4 (here, as in Fig. 1, the neighbourhood of the step is the mirror image in the x axis). The ordinates of the initial points of the integral curves, which in Fig. 4 are given the numbers 1, 2, ..., 6, had the values (the abscissa ξ is fixed)

$$\xi = \xi_0 = -0.5, \quad \eta_n^{(0)} = 0.25n \quad (n = 1,2,\ldots,6)$$

It can be seen from Fig. 4 that the directions of the trajectories σ_1 in the vicinity of the step surface are, on the whole, parallel to the step contour and there are deviations beyond the limits of the inclined face of the step. This enables us to assume that the force $\mathbf{F}_z$ gives rise to cracks on the surface of a step made of a brittle material, the direction of which is perpendicular to the step contour.

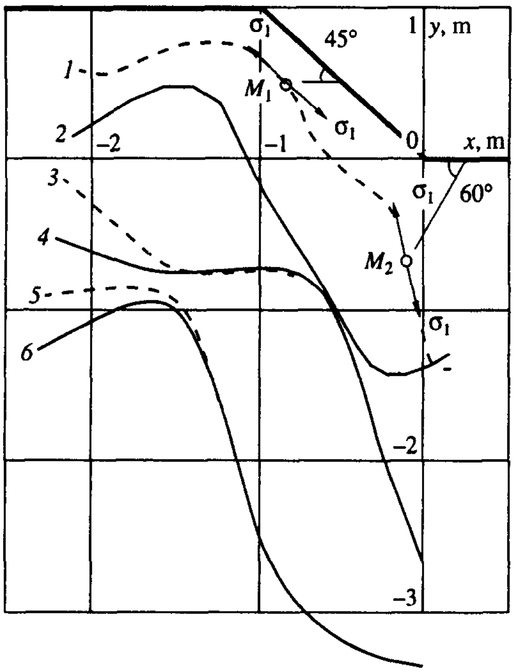

Fig. 4

The principal stresses σ_1, as well as the directions of the maximum shear stresses τ_{max} are shown at the two points (M_1 and M_2) on the trajectory emerging from the initial point ($\xi_0 = -0.5$, $\eta_1^{(0)} = 0.25$). The directions of τ_{max} at the point M_1, which is located in the vicinity of the inclined face of the step, make an angle of about 45° with it, and the directions of τ_{max} at the point M_2, which is located in the vicinity of the lower corner of the step, make an angle of about 60° with its lower face. This enables us to postulate that the force $\mathbf{F}_z$ gives rise to shears on the surface of a step made of a plastic material, the direction of which is parallel to the direction of action of the force.

Hence, in the case of steps made of a brittle material, a concentrated force can lead to the occurrence of cracks which are directed perpendicular to the contour of the step while, in the case of steps made of a plastic material, it can lead to the appearance of shears in the direction of action of the force.

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